2017-2018 年度美国"大联盟"(Math League)思维探索第一阶段活动

(六年级)

(活动日期：2017 年 11 月 26 日，答题时间：90 分钟，总分 200 分)

学生诚信协议：答题期间，我确定没有就所涉及的问题或结论，与任何人、用任何方式交流或讨论，我确定我所填写的答案均为我个人独立完成的成果，否则愿接受本次成绩无效的处罚。

选择题：每小题 5 分，答对加 5 分，答错不扣分，共 200 分。

1. Pick any integer greater than 1. Double it twice, then triple the result. The final outcome is __?__ of your starting integer.

   A) 700%   B) 1100%   C) 1200%   D) 1300%

2. Barry listened to the radio for 3 hours and 36 minutes. Rounded to the nearest 10 minutes, for how many minutes was Barry listening?

   A) 210   B) 220   C) 330   D) 340

3. Divide 99 by 22 to get a quotient and remainder. Divide that remainder by that quotient, and the new remainder is

   A) 4   B) 3   C) 2   D) 1

4. A man had five pieces of chain, each made up of three links, figure below. He wanted to join the five pieces together to make a big chain of fifteen links and went to a blacksmith to see how much it would cost. "Well," said the blacksmith, "I will charge you 50 cents for cutting a link and $1.00 for welding a link. Any bending that is required is free." Given those prices, what is the smallest amount of money for which the job could be done?

   Note: In a chain, each link is connected to one or two other links.

   A) $4.50   B) $5.00   C) $5.50   D) $6.00

5. A bee sat on the head of a horse rider whose horse was trotting eastbound at a steady five miles per hour. Some distance ahead on the same path, another horse and rider were approaching westbound, also at five miles per hour. When the two horses were 20 miles apart, the bee left the first horse rider and flew toward the second horse at a rate of ten miles per hour. Upon reaching the second horse, the bee immediately turned around and flew back at the same rate to the first horse. If the bee kept up this performance until the two riders met, how far (in miles) did he travel from the moment he left the first horse rider?

   A) 10   B) 20   C) 30   D) None of the above
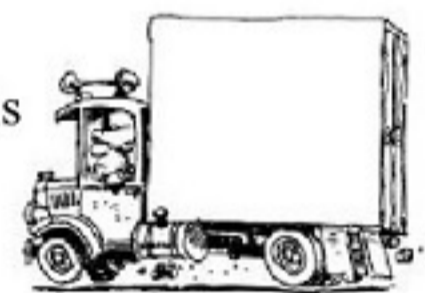
6. Which of the following is the sum of the prime factors of 2018?

   A) 11   B) 219   C) 1011   D) 2019

7. If the length of the longest side of a triangle is 18, which of the following could *not* be the length of its second-longest side?

   A) 9   B) 10   C) 12   D) 17

8. My final score in a competition is the average of my scores on 5 rounds. To get a final score of 88 after getting 84, 80, and 92 on the first 3 rounds, what must be my average score for the last 2 rounds?

    A) 88   B) 90   C) 92   D) 96

9. Mr. Rice had breakfast one day at a restaurant with Mr. Wheat. When it came time to pay the bill, it was found that Mr. Rice had as many one-dollar bills as Mr. Wheat had quarters. (Mr. Rice had one-dollar bills only, and Mr. Wheat had quarters only.) Rather than each man paying separately, Mr. Rice paid his share of the bill, $6, to Mr. Wheat. At that point, Mr. Wheat had four times as much money as Mr. Rice. How much money did Mr. Rice have at the beginning?

    A) $6   B) $8   C) $9   D) $12

10. Professor Peach teaches chemistry to clever kids. The ratio of freshmen to other students in his class is 3:8. The total number of students in Professor Peach's class could be

    A) 42   B) 45   C) 56   D) 77

11. $4^{40} \div 2^{20} =$

    A) 2^2 B) 2^4 C) 2^{20} D) 2^{60}

12. Mr. Bogsworth once left a will which read:
    To Bob, twice as much as to Betty.
    To Brian, twice as much as to Bob.
    To Bill, twice as much as to Brian.
    If his estate was valued at $45000, how much money did Betty, one of his four heirs, receive?

    A) $1000   B) $2000   C) $3000   D) $6000

13. I paid $5 and got 5 quarters, 5 dimes, and 5 nickels in change. I spent

    A) $3.00   B) $3.25   C) $3.45   D) $3.75

14. One side of Todd's truck is a perfect rectangle with an area of 12 m². If its length is 3 times its width, then its perimeter is

    A) 8 m   B) 12 m   C) 16 m   D) 20 m

15. If a bird in the hand is worth two in the bush, and a bird in the bush is worth four in the sky, then 4 birds in the hand are worth __?__ birds in the sky.

    A) 1   B) 4   C) 16   D) 32

16. On each of the four shelves of my bookcase is a different prime number of books. There could be a total of __?__ books on my shelves.

    A) 15   B) 21   C) 22   D) 24

17. Seven years ago I realized that my age would be tripled twelve years from then. How old am I now?

    A) 11   B) 13   C) 16   D) 18

18. How many fractions with a numerator of 1 and a whole-number denominator are greater than 0.01 and less than 1?

    A) 98   B) 99   C) 100   D) 101

19. If I write the letters R-E-P-E-A-T repeatedly, stopping when I have written exactly 100 letters, how many times do I write the letter E?

    A) 16   B) 18   C) 32   D) 34

20. On my map, 1 cm represents 100 km. If a park shown on the map is a rectangle that is 2.5 cm by 4 cm, the area of the actual park is __?__ km².

   A) 100   B) 1000   C) 10 000   D) 100 000

21. Gloomy Gus's Tuesday rain cloud shows up every Tuesday at 8:30 A.M. and every 50 minutes after that. Its last appearance on Tuesday is at __?__ P.M.

   A) 11:00   B) 11:10   C) 11:30   D) 11:50

22. If my lucky number divided by its reciprocal is 100, then the square of my lucky number is

   A) 100   B) 10   C) 1   D) $\dfrac{1}{100}$

23. A boy and his sister were walking down the street one afternoon when they met a kind old man. When the old man asked them about the size of their family, the boy quickly answered. "I have as many brothers as I have sisters," he proudly stated. Not to be left out, the girl added, "I have three times as many brothers as I have sisters." Can you tell how many boys and girls in total there were in their family?

   A) 5   B) 6   C) 7   D) 8

24. A farmer was asked how many pigs he had. "Well," he said, "if I had just as many more again, plus half as many more, plus another 1.5 times more, I would have three dozen." How many pigs did he have?

   A) 6   B) 9   C) 12   D) 15

25. 15% of 80 is 40% of

   A) 30   B) 55   C) 105   D) 210

26. It took me 90 minutes to cycle 45 km to the beach. Later I got a ride from the beach to the park at twice my cycling speed. If the ride to the park took 15 minutes, what distance did I travel from the beach to the park?

   A) 15 km   B) 30 km   C) 45 km   D) 135 km

27. Ted, Rick, and Sam painted a wall together. Ted painted 80% more of the wall than Sam painted. Sam painted 40% less than Rick. Ted painted __?__ of the amount that Rick painted.

   A) 102%   B) 108%   C) 120%   D) 140%

28. The greatest integer power of 20 that is a divisor of 50^{50} is

   A) 20^{20} B) 20^{25} C) 20^{50} D) 20^{125}

29. The ones digit of the sum of all even integers from 2 to 1492 is

   A) 2   B) 4   C) 8   D) 0

30. The median of $\dfrac{1}{2}$, $\dfrac{1}{3}$, $\dfrac{1}{4}$, $\dfrac{1}{5}$, $\dfrac{1}{6}$, and $\dfrac{1}{7}$ is

   A) $\dfrac{1}{8}$ B) $\dfrac{223}{840}$ C) $\dfrac{5}{14}$ D) $\dfrac{9}{40}$

31. The average of all positive even integers from 2 to 2018 is
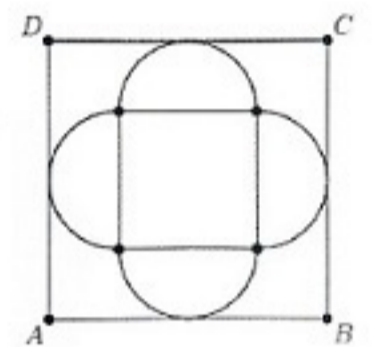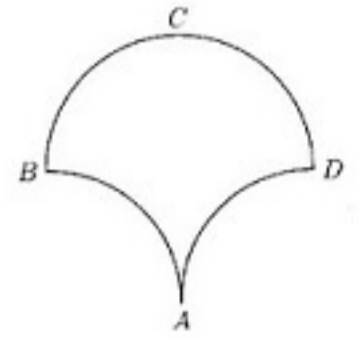
   A) 1000   B) 1009   C) 1010   D) 1014

32. Pirate Percy has 300 coins in his chest. Of the Spanish coins, 20% are gold. If 100 of the coins are gold but not Spanish and 70 of the coins are neither gold nor Spanish, how many Spanish gold coins are in Percy's chest?

   A) 20   B) 26   C) 30   D) 34

33. When I divided the population of my city by the number of streets in the city, I got a remainder of 18. If the exact quotient on my calculator was 123.06, how many streets are there in my city?

 A) 60 B) 120 C) 186 D) 300

34. What is the greatest number of 3-by-7 rectangles that can be placed inside an 80-by-90 rectangle with no overlapping?

 A) 312 B) 330 C) 334 D) 342

35. How many four-digit whole numbers have four different even digits and a ones digit greater than its thousands digit?

 A) 36 B) 54 C) 60 D) 90

36. Both arcs AB and AD are quarter circles of radius 5, figure on the right. Arc BCD is a semi-circle of radius 5. What is the area of the region $ABCD$?

 A) 25 B) $10 + 5\pi$ C) 50 D) $50 + 5\pi$

37. In the figure on the right, the side-length of the smaller square is 4. The four arcs are four semi-circles. Each side of square $ABCD$ is tangent to one of the semi-circles. The area of $ABCD$ is

 A) 32 B) 36 C) 48 D) 64

38. A million is a large number, a "1" followed by 6 zeros. A googol is a large number, a "1" followed by one hundred zeros. A googolplex is a large number, a "1" followed by a googol of zeros. A googolplexian is a large number, a "1" followed by a googolplex of zeros. A googolplexian is

 A) 10^{10} B) $10^{10^{100}}$ C) $10^{10^{10^{100}}}$ D) none of the above

39. If the total number of positive integral divisors of n is 12, what is the greatest possible total number of positive integral divisors of n^2?

 A) 23 B) 24 C) 33 D) 45

40. Of all the isosceles triangles whose perimeter is 20 and whose side-lengths are integers, what is the length of the base of the triangle with the largest area?

 A) 2 B) 5 C) 6 D) 8

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六年级试卷答案

题号	1	2	3	4	5	6	7	8	9	10
答案	C	B	B	A	B	C	A	C	B	D
题号	11	12	13	14	15	16	17	18	19	20
答案	D	C	A	C	D	B	B	A	D	D
题号	21	22	23	24	25	26	27	28	29	30
答案	C	A	A	B	A	A	B	B	A	D
题号	31	32	33	34	35	36	37	38	39	40
答案	C	B	D	D	A	C	D	C	D	C